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FINITE ELEMENT ANALYSIS OF TRANSONIC FLOWS.(U)
MAY 80 G J FIX DAAG29-77-G-0026

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The work reported here centered around the development of finite element algorithms for the following class of problems:

1. Transonic flow problems governed by the potential equations
2. Moving boundary problems
3. Incompressible flow problems governed by the Navier-Stokes equations.

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Final Report To

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on

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The work supported by this contract centered around the development of finite element algorithms for the following class of problems:

1. Transonic flow problems governed by the potential equations
2. Moving boundary problems
3. Incompressible flow problems governed by the Navier-Stokes equations.

<u>Work on the first problem</u>

The first major result appeared in reference [1] in the list of papers published during the contract. This paper contains a new variational formulation of transonic flow problems as well as calculations from sample flow problems. These numerical examples indicated that the method yielded second order accuracy in the velocity field if linear elements were used. Further refinement of the algorithm was introduced in reference [4] where a reformulation of the basic variational principle was developed using least square ideas. Sample numerical experiments indicated that it retained the optimal accuracy yet required the inversion of a far simplier matrix in order to compute the fluid velocity field. Subsequent work involved mathematical analysis of these methods in order to determine when optimal accuracy can be expected and to find other strong and weak points of the methods. This work was reported in references [4] - [8], [12] - [14]. It was found

that if a "dash pot" treatment of shocks was used, then second order accuracy could be achieved in general. Moreover, the algebraic system could be solved with roughly the same amount of arithmetic operations as the first order finite difference schemes in current use. Thus from the point of view of numerical analysis, these methods introduce an order of magnitute increase in efficiency over existing methods.

The major drawback of the methods is that they are extremely hard to program. This may be a prohibitive limitation to small research groups with limited computational equipment and limited experience with finite element methods. However, larger groups with experience with finite element methods should be able to capture the order of magnitude increase in efficiency provided by these new methods.

Work on the third problem

During the analysis of the new methods introduced for transonic flow problemsit became clear that these results applied also to the standard and widely used finite element approximations to other flow problems. The most interesting result in this regard concerns the Navier-Stokes equations. Our analysis indicated that if one used arbitrary finite element spaces in the approximation, then instabilities could arise. Only select spaces will work, and conditions are given with which the suitability of a particular space can be checked. This in part explains the wide divergence of opinion concerning the suitability of finite elements for flow

problems. The fact that a given finite element would work with a particular finite element space, and not with a closely related one was not anticipated from the use of the method for problems in solid mechanics.

Work on the second problem

This was work done in the last part of the contract and was reported in references [6] and [17] with the latter being the more important paper. The object was to construct numerical algorithms capable of dealing with two phase problems where effects of supercooling (or superheating) were present. The enthalpy method widely used for Stefan problems is not directly applicable here since the enthalpy in effect becomes multivalued in regions of supercooling. To avoid these problems an alternate phase field model is introduced where the phase is an independent variable satisfying a nonlinear parabolic partial differential equation. Pilot numerical experiments indicated that the method had some promise, yet several computational problems need to be resolved, the most important of which is the fact that the equation for the phase is much stiffer than the diffusion equation for the temperature field. Thus different grids with varying length scales are required for the temperatures and phase. Intuition has suggested some reasonable choices, however a thorough analysis is required before a truly practical algorithm will result.

List of Publications During Contract

* 1. (with M. Gurtin), "On patched variational methods", Numer. Math., 28(1977), 259-271.

* 2. (with B. Neta), "Finite element approximation of a nonlinear diffusion problem", Comp. and Math. with Appls. 3(1977), 287-298.

* 3. (with M. Gunzburger), "Downstream Boundary Conditions for viscous flow problems", Comp. and Math. with Appls. 3(1977), 53-63.

* 4. (with M. Gunzburger), "On least squares approximation to indefinite problems of the mixed types", Int'l. Jour. for Numer. Mtd. in Eng., 12(1978), 453-470.

5. (with S. Marin), "Variational Methods for underwater acoustic problems", J. of Comp. Phy., 28(1978), 1-18.

* 6. "Numerical Methods for Alloy Solidification Problems", Moving Boundary Problems, Ward, ed., Academic Press, 1978.

* 7. "Variational Methods for Elliptic Boundary Value Problems", A.M.S. Short Course on Numerical Analysis, Oliger, ed., A.M.S. publications, 1978.

* 8. (with M. Gunzburger and R. A. Nicolaides), "Least squares finite element methods", NASA-ICASE Report 77-18, revised version published in Comp. and Math. with Appls., 5 (1979), 87-98.

* 9. (with M. Gunzburger and R. A. Nicolaides), "On mixed finite element methods", NASA-ICASE Reports 77-17 and 78-7; revised version submitted to Numer. Math.

10. (with M. Gunzburger), "On numerical methods for acoustic problems", NASA-ICASE Report 78-15, revised version accepted for publication in Math. and Computers with Applications, 1979.

* 11. (with C. Coffman), "Constructive Approaches to Mathematical Models", Proceedings of a symposium in honor of R. J. Duffin, Academic Press, 1979.

* 12. (with M. Gunzburger and R. A. Nicolaides), "Theory and applications of mixed finite element methods", to appear in reference [33], 1979.

(continued)

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* 13. (with M. Gunzburger and R. A. Nicolaides), "On mixed finite element methods for a class of nonlinear boundary value problems", to appear T.I.C.O.M. Conference on Nonlinear Mechanics, T. Oden (ed.), 1979.

* 14. "On the structure of errors in mixed finite element methods", submitted to R.A.I R.O., 1979.

* 15. (with H. E. Donley), "On Finite Element Approximations for a Class of Ill-Posed Problems", accepted for publication, SIAM Jour. Numerical Analysis.

* 16. "Finite Element Approximations to Flow Problems," to appear, Third International Conference on Finite Elements in Water Resources, May 1980.

* 17. "Phase Field Methods for Free Boundary Value Problems," to appear, Second International Symposium on Innovative Numerical Analysis in Applied Engineering Science, University of Virginia Press.

18. (with R. A. Nicolaides), "An Analysis of Mixed Finite Element Approximations for Periodic Acoustic Wave Propagation," to appear, SIAM Jour. Numerical Analysis.

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Personnel Supported

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